

Evolution of the Longitudinal Structure Function at Small- x

G.R.Boroun*

Physics Department, Razi University, Kermanshah 67149, Iran

(Dated: February 5, 2014)

We derive an approximation approach to evolution of the longitudinal structure function, by using a Laplace-transform method. We solve the master equation and derive the longitudinal structure function as a function of the initial condition $F_L(x, Q_0^2)$ at small- x . Our results are independent of the longitudinal coefficient functions and extend from the leading-order (LO) up to next-to-next-to-leading order (NNLO). The comparisons with H1 data and other parameterizations are made and results show that they are in agreement with H1 data and some phenomenological models.

1 Introduction

The measurement of the longitudinal structure function $F_L(x, Q^2)$ is of great theoretical importance, since it may allow us to distinguish between different models describing the QCD evolution at small- x . In deep-inelastic scattering (DIS), the structure function measurements remain incomplete until the longitudinal structure function F_L is actually measured [1]. The longitudinal structure function in deep inelastic scattering is one of the observable from which the gluon distribution can be unfolded.

At small- x values, the dominant contribution to $F_L(x, Q^2)$ comes from the gluon operators. Hence a measurement of $F_L(x, Q^2)$ can be used to extract the gluon structure function and therefore the measurement of F_L provides a sensitive test of perturbative QCD [2,3]. At this region, the longitudinal structure function can be related to the gluon and sea-quark distribution. In principle, the data on the singlet part of the structure function F_2 constrain the sea quarks and the data on the slope $\frac{dF_2}{d\ln Q^2}$ determine the gluon density. Moreover, the longitudinal structure function $F_L(x, Q^2)$ can be related at small- x with structure function F_2 and the derivation $dF_2/d\ln Q^2$ [4-9]. In this way most precise predictions based on data of F_2 and $dF_2/d\ln Q^2$ can be obtained for F_L . These predictions can be considered as indirect experimental data for F_L .

The behavior of the structure function F_2 at small x and large Q^2 have been discussed considerably over the past years [10] into the double asymptotic scaling. Double asymptotic scaling follows from a computation [11-12] of the asymptotic form of the structure function $F_2^p(x, Q^2)$ at small x based on the use of the operator product expansion and renormalization group at leading perturbative order. It thus relies only on the assumption that any increase in $F_2^p(x, Q^2)$ at small x is generated by perturbative QCD evolution, rather than being due

to some other (nonperturbative) mechanism manifested by an increase in the starting distribution $F_2^p(x, Q_0^2)$. A better understanding of QCD physics at small x have been achieved by considering more observables and thus (over-)constraining the definite of the parton densities [13]. In Ref.13 the interplay between perturbative and non-perturbative dynamics has been pointed out in the context of QCD analysis of the small x behavior of the proton structure function $F_2^p(x, Q^2)$. The factorization theorem of mass singularities provides a representation of F_2 in terms of phenomenological parton densities and perturbatively computable splitting and coefficient functions. Then, S.Catani have been considered the physical anomalous dimensions relating the singlet components of F_2 and F_L . That solve it for Mellin transformations of the parton densities. In N-space, an available program that deal with DGLAP evolution is QCD PEGASUS [14], which is a parton distribution functions (PDFs) evolution program based on Mellin-space inversion.

To study the longitudinal structure function we use the Laplace-transform technique for solving the Altarelli-Martinelli equation[15]. In recent years, Laplace-transform technique have proved to be valuable tools for the solving of the DGLAP [16-18] evolution equations in the LO approximation up to NLO [19-23]. Here, a similar procedure is used to derive evolution of the longitudinal structure function inside the proton. We obtain an analytical solution for evolution of the longitudinal structure function at small x in terms of the initial condition at the starting scale Q_0^2 . Thus we can determine the longitudinal structure function at small- x directly as a function of the initial longitudinal structure function, and this result is independent of the knowledge about the coefficient functions for F_L at LO up to NNLO.

The content of our paper is as follows. In section 2 we describe the basic theory to extract the longitudinal structure function from the gluon distribution function at small- x . Section 3 is devoted to the analytical solution of the master equation for the longitudinal structure function by Laplace-transform technique. Finally, an analytical analysis of our solution is presented and the obtained results are compared with other methods which

*boroun@razi.ac.ir; grboroun@gmail.com

are followed by results and discussions.

2. Basic Theory

We specifically consider the longitudinal structure function F_L , projected from the hadronic tensor by combination of the metric and the spacelike momentum transferred by the virtual photon ($g_{\mu\nu} - q_\mu q_\nu / q^2$). As it is proportional to hadronic tensor as follows

$$F_L(x, Q^2)/x = \frac{8x^2}{Q^2} p_\mu p_\nu W_{\mu\nu}(x, Q^2), \quad (1)$$

where $p^\mu (p^\nu)$ is the hadron momentum and $W^{\mu\nu}$ is the hadronic tensor. In this relation we neglecting the hadron mass.

The basic hypothesis is that the total cross section of a hadronic process can be written as the sum of the contributions of each parton type (quarks, antiquarks, and gluons) carrying a fraction of the hadronic total momentum. In the case of deep-inelastic scattering it reads

$$d\sigma_H(p) = \sum_i \int dy d\hat{\sigma}_i(y, p) \Pi_i^0(y), \quad (2)$$

where $d\hat{\sigma}_i$ is the cross section corresponding to the parton i and $\Pi_i^0(y)$ is the probability of finding this parton in the hadron target with the momentum fraction y . Now, taking into account the kinematical constraints one gets the relation between the hadronic and the partonic structure functions

$$\begin{aligned} f_j(x, Q^2) &= \sum_i \int_x^1 \frac{dy}{y} f_j\left(\frac{x}{y}, Q^2\right) \Pi_i^0(y) \\ &= \sum_i f_j \otimes \Pi_i^0(y), \quad j = 2, L, \end{aligned} \quad (3)$$

where $f_j(x, Q^2) = F_j(x, Q^2)/x$ and the symbol \otimes denotes convolution according to the usual prescription, $f(x) \otimes g(x) = \int_x^1 \frac{dy}{y} f(y) g(\frac{x}{y})$. Equation (3) expresses the hadronic structure functions as the convolution of the partonic structure function, which are calculable in perturbation theory, and the probability of finding a parton in the hadron which is a nonperturbative function. At small values of x , F_L is driven mainly by gluons through the transition $g \rightarrow q\bar{q}$ ($g(x, Q^2)$ is the gluon density). Recently [24], the process $gg \rightarrow gg$ where the external gluons are on-shell have been obtained by calculation of the forward jet vertex at next-to-leading order in the BFKL formalism, offering an explicit derivation of the gluon-initiated contribution. As this adds to that previously [25] calculated for the quark-initiated vertex and completes the derivation of the full vertex.

Therefore F_L can be used for the extraction of the gluon distribution in the proton, and it provides a crucial test

of the validity of perturbative QCD in this kinematical range. So, in correspondence with Eq.(3) one can write Eq.(1) for the gluon density dominated at small- x values by follows

$$F_L^g/x = \frac{\alpha_s}{4\pi} [f_{L,G}^{(LO+\dots)} \otimes g^0], \quad (4)$$

where $f_{L,G}$ s are the LO up to NNLO partonic longitudinal structure function corresponding to gluons [26-27]. We present the expressions, after full agreement has been achieved, in the form of kernels $C_{L,G}$ which give F_L upon convolution with the gluon distribution

$$\begin{aligned} F_L^g(x, Q^2) &= \frac{\alpha_s(Q^2)}{4\pi} \langle e^2 \rangle C_{L,G}(\alpha_s, \frac{x}{y}) \otimes G(y, Q^2) \\ &= \frac{\alpha_s(Q^2)}{4\pi} \langle e^2 \rangle \int_x^1 \frac{dy}{y} C_{L,G}(\alpha_s, \frac{x}{y}) G(y, Q^2), \end{aligned} \quad (5)$$

where $C_{L,G}(x, Q^2)$ is the DIS coefficient function for $F_L^g(x, Q^2)$. The average squared charge ($=5/18$ for even n_f) is represented by $\langle e^2 \rangle$, where n_f denotes the number of effectively massless flavours. We write the perturbative expansion of the coefficient functions as

$$\begin{aligned} C_{L,G}(\alpha_s, x) &= c_{L,G}^{\text{LO}}(x) + \frac{\alpha_s(Q^2)}{4\pi} c_{L,G}^{\text{NLO}}(x) \\ &\quad + \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 c_{L,G}^{\text{NNLO}}(x). \end{aligned} \quad (6)$$

The longitudinal coefficient functions are given in Refs.[28-30]. The running coupling constant $\frac{\alpha_s}{4\pi}$ has the form in the LO, NLO and NNLO respectively [31]

$$\frac{\alpha_s^{\text{LO}}}{4\pi} = \frac{1}{\beta_0 t}, \quad (7)$$

$$\frac{\alpha_s^{\text{NLO}}}{4\pi} = \frac{1}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} \right], \quad (8)$$

and

$$\begin{aligned} \frac{\alpha_s^{\text{NNLO}}}{4\pi} &= \frac{1}{\beta_0 t} \left[1 - \frac{\beta_1 \ln t}{\beta_0^2 t} + \frac{1}{(\beta_0 t)^2} \left[\left(\frac{\beta_1}{\beta_0} \right)^2 \right. \right. \\ &\quad \left. \left. (\ln^2 t - \ln t + 1) + \frac{\beta_2}{\beta_0} \right] \right]. \end{aligned} \quad (9)$$

where $\beta_0 = \frac{1}{3}(33 - 2n_f)$, $\beta_1 = 102 - \frac{38}{3}n_f$ and $\beta_2 = \frac{2857}{6} - \frac{6673}{18}n_f + \frac{325}{54}n_f^2$ are the one-loop, two-loop and three-loop corrections to the QCD β -function. The variable t is defined as $t = \ln(\frac{Q^2}{\Lambda^2})$ and Λ is the QCD cut-off parameter.

Therefore the master equation for the longitudinal structure function can be written as

$$\begin{aligned} \left(\frac{\alpha_s}{4\pi} \langle e^2 \rangle \right)^{-1} F_L^g(x, Q^2) &\equiv \mathcal{F}_L^g(x, Q^2) \\ &= \int_x^1 \frac{dy}{y} C_{L,G}(\alpha_s, \frac{x}{y}) G(y, Q^2). \end{aligned} \quad (10)$$

This equation for the longitudinal structure function has the explicit dependence to the gluon distribution function. To extract an approximation solution of the longitudinal structure function evolution from the DGLAP equation without dependence to the gluon distribution, we should solve Eq.10 for the evolution of the longitudinal structure function $F_L(x, Q^2)$ into $F_L(x, Q_0^2)$. Let us consider the differential form of Eq.5 as for simplicity we assume that the running coupling constant in $C_{L,G}(\alpha_s, x)$ is constant at NLO up to NNLO. Therefore we have

$$\frac{\partial F_L^g}{\partial \ln Q^2} = \frac{\langle e^2 \rangle}{4\pi} \frac{d\alpha_s}{d\ln Q^2} C_{L,G}\left(\frac{x}{y}\right) \otimes G(y, Q^2) + \frac{\alpha_s}{4\pi} \langle e^2 \rangle C_{L,G}\left(\frac{x}{y}\right) \otimes \frac{\partial G(y, Q^2)}{\partial \ln Q^2}, \quad (11)$$

where the derivative of the gluon distribution with respect to $\ln Q^2$ i.e. $\partial G(x, Q^2)/\partial \ln Q^2$, at small- x is given by the DGLAP evolution equation as we have [15-18, 32-35]

$$\frac{\partial G(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{4\pi} \int_x^1 \frac{dy}{y} [P_{gg}\left(\frac{x}{y}, \alpha_s(Q^2)\right) G(y, Q^2)], \quad (12)$$

where the splitting functions P_{ij} 's are the LO, NLO and NNLO Altarelli-Parisi splitting kernels as

$$P_{gg}(x, \alpha_s(Q^2)) = P_{gg}^{\text{LO}}(x) + \frac{\alpha_s(Q^2)}{4\pi} P_{gg}^{\text{NLO}}(x) + \left(\frac{\alpha_s(Q^2)}{4\pi}\right)^2 P_{gg}^{\text{NNLO}}(x). \quad (13)$$

3. Master equation for evolution of the longitudinal structure function at small- x

To evolution of the longitudinal structure function, we follow the procedure that was used by authors Refs.[19-22] and employ the Laplace-transform method to solve Eqs.10-12. Now we use the coordinate transformation as

$$v \equiv \ln(1/x). \quad (14)$$

In v -space, Eq.10 appears as

$$\hat{\mathcal{F}}_L^g(v, Q^2) = \int_0^v \hat{C}_{L,G}(v-w) \hat{G}(w, Q^2) dw, \quad (15)$$

where the functions $\hat{\mathcal{F}}_L^g$, $\hat{C}_{L,G}$ and \hat{G} are given by

$$\begin{aligned} \hat{\mathcal{F}}_L^g(v, Q^2) &\equiv \mathcal{F}_L^g(e^{-v}, Q^2), \\ \hat{C}_{L,G}(v, Q^2) &\equiv C_{L,G}(e^{-v}, Q^2), \\ \hat{G}(v, Q^2) &\equiv G(e^{-v}, Q^2). \end{aligned} \quad (16)$$

If we take the Laplace-transform of Eq.15, then we have

$$\begin{aligned} \mathcal{L}[\hat{\mathcal{F}}_L^g(v, Q^2); s] \\ = \mathcal{L}\left[\int_0^v \hat{C}_{L,G}(v-w) \hat{G}(w, Q^2) dw; s\right]. \end{aligned} \quad (17)$$

Therefore

$$F_L^g(s, Q^2) = h(s)g(s, Q^2). \quad (18)$$

Also, derivative of the longitudinal structure function (Eq.11) in v -space appears as

$$\frac{\partial F_L^g(s, Q^2)}{\partial \ln Q^2} = \frac{d\ln \alpha_s}{d\ln Q^2} F_L^g(s, Q^2) + \frac{\alpha_s}{4\pi} F_L^g(s, Q^2) \Phi_g(s) \quad (19)$$

At small- x derivative of the gluon distribution function (Eq.12) in v -space is straightforward [22], as we can be written this equation by this form

$$\frac{\partial g(s, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{4\pi} \Phi_g(s) g(s, Q^2), \quad (20)$$

where $g(s) = \mathcal{L}[\hat{G}(v); s] = \int_0^\infty \hat{G}(v) e^{-sv} dv$ ($\hat{G}(v) \equiv G(e^{-v})$). The coefficient function $\Phi_g(s)$ at LO is given by [22]

$$\begin{aligned} \Phi_g^{\text{LO}}(s) = \frac{33-2n_f}{3} + 12\left(\frac{1}{s} - \frac{2}{1+s} - \frac{1}{2+s}\right) \\ - \frac{1}{3+s} - \psi(1+s) - \gamma_E \end{aligned} \quad (21)$$

where $\psi(x)$ is the digamma function and $\gamma_E = 0.5772156\dots$ is Euler's constant.

For obtain an approximation form for the evolution of the longitudinal structure function at small- x , we rewrite Eq.19 in s -space as

$$\frac{\partial \ln F_L^g(s, Q^2)}{\partial \ln Q^2} = \frac{d\ln \alpha_s}{d\ln Q^2} + \frac{\alpha_s}{4\pi} \Phi_g(s). \quad (22)$$

In the above equation we take the inverse Laplace transform using the known inverse $\mathcal{L}^{-1}[F(s, Q^2); v] = \hat{F}(v, Q^2)$, we find that

$$\frac{\partial \ln \hat{F}_L^g(v, Q^2)}{\partial \ln Q^2} = \frac{d\ln \alpha_s}{d\ln Q^2} \delta(v) + \frac{\alpha_s}{4\pi} \hat{\Phi}_g(v), \quad (23)$$

where $\mathcal{L}^{-1}[\Phi_g(s); v] = \hat{\Phi}_g(v) = \Phi_G(x)$.

The solution of the evolution equation of the longitudinal structure function in terms of the initial values of function $F_L(x, Q_0^2)$ is straightforward. Finally we have

$$F_L^g(x, Q^2) = F_L(x, Q_0^2) \eta(Q^2, Q_0^2) e^{\tau(Q^2, Q_0^2) \Phi_G(x)}, \quad (24)$$

where

$$\eta(Q^2, Q_0^2) = \frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}, \quad (25)$$

and

$$\tau(Q^2, Q_0^2) = \frac{1}{4\pi} \int_{Q_0^2}^{Q^2} \alpha_s(Q'^2) d\ln Q'^2. \quad (26)$$

This result is an approximation approach to the evolution equation for the longitudinal structure function and gives an analytical expression for the evolution of F_L at leading order (LO). We emphasize that $F_L^g(x, Q^2)$ directly is not dependence to the gluon distribution function and to the longitudinal coefficient function. It is dependence to the running coupling constant and to the gluonic splitting function at LO up to NNLO, as the explicit form of the gluonic splitting function at LO ($\phi_G^{LO}(x)$) is

$$\Phi_G^{LO}(x) = \frac{33 - 2n_f}{3} + 12(1 - 2x + x^2 - x^3 - \gamma_E) + 6x(1 + \coth(\frac{1}{2}\ln\frac{1}{x})). \quad (27)$$

This method can be generalized to NLO up to NNLO. The evolution of the NLO (up to NNLO) splitting coefficients is straightforward, but the method can not be completely extended because of the impossibility of analytically inverting the required Laplace transform to the NLO (up to NNLO) splitting functions needed in the DGLAP evolution equation (Eq.12) [22]. At the limit of small- x the two and three-loop splitting functions read [36-38]

$$\Phi_G^{NLO}(x) \rightarrow 4 \left(\frac{12C_F T_F - 46C_A T_F + C_A^2(41 - 3\pi^2)}{9} \right) \quad (28)$$

and

$$\Phi_G^{NNLO}(x) \rightarrow (14214.2 + n_f 182.958 - n_f^2 2.79835) - (2675.85 + n_f 157.269) \ln(\frac{1}{x}) \quad (29)$$

with $C_A = N_c = 3$, $C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$ and $T_F = \frac{1}{2}n_f$.

4. Results and Discussions

In this paper, we have obtained an analytical solution for the evolution of the longitudinal structure function at small- x . Our solution is model independent of the gluon distribution function and it is free of any longitudinal coefficient function. It is only dependence on the running coupling constant and gluonic splitting function based on the Laplace transform technique at small- x . It can be provided as our results at small- x can predict be the longitudinal structure function to high- Q^2 values and suggest that the precise measurement of experimental values for $F_L(x, Q^2)$ over a wide kinematic range of small- x and high- Q^2 can be done (Recently the longitudinal structure function measured by H1-2013 Collaboration [46-48] at high- Q^2 values). To confirm the method and results, the calculated values are compared with the H1 data on the longitudinal structure function. It is shown that, our results are in agreement with experimental H1 data for F_L , if one takes into the

total errors, and is consistent with a higher order QCD calculations of F_L which essentially show increase as x decreases. We observe that the calculations results are consistent with the two pomeron model. Thus implying that Regge theory and perturbative evolution may be made compatible at small- x . The result not only gives striking support to the two- pomeron description of small x behavior, but also a rather clean test of perturbative QCD itself.

We computed the predictions for the longitudinal structure function in the kinematic range where it has been measured by H1 collaboration [4,6-7,46-48] and compared with DL model [39-42] based on hard Pomeron exchange and also k_T factorization [49] at small x . At small- x , the longitudinal structure function receive large logarithmic corrections coming from re-summation of large powers of $\alpha_s \ln \frac{1}{x}$, where goes beyond the standard collinear factorization formalism [50-51] using the unintegrated gluon density obtained from the Kwiecinski-Martin-Stasto (KMS) approach [52]. This approach includes important effects of higher order resummation. Using k_T factorization at the on-shell limit which the transverse momentum of the gluon k^2 is much smaller than the virtuality of the photon, $k^2 \ll Q^2$ and this is consistent with the collinear factorization. The k_T factorization formula can be determined the inclusive cross section in dipole representation. Where, the longitudinal structure function is proportional to the longitudinal polarized photon-proton cross section, or it is proportional to the color dipole cross section [49,53]. With respect to the GBW saturation model [54-55] for the dipole cross section, the leading twist terms are proportional to the linear terms in the gluon density. Consequently, the leading twist-2 part in the dipole picture gives the longitudinal structure function as derived in the Refs.[53-55].

Our analytical predictions are presented as functions of x for the $Q^2 = 20, 45$ and 200 GeV^2 . Here we obtain the gluonic longitudinal structure function F_L^g by evolving up in Q^2 from $F_L(x, Q_0^2)$ at the input scale, $Q_0^2 = 1 \text{ GeV}^2$, obtained in an approximation analysis to the DGLAP evolution equation with the DL initial starting function. We defined the coupling via the $n_f = 4$ definition of Λ_{QCD} for the MRST set of partons[43-45] as the values of Λ_{QCD} at LO up to NNLO is displayed in Table 1.

The results are presented in Figs.1-3 where they are compared with the recent H1 data [46-48] and with the results obtained with the help of other standard gluon distribution functions. As can be seen in all figures, the increase of our calculations for the longitudinal structure functions $F_L^g(x, Q^2)$ towards small- x are consistent with the NLO QCD calculations, reflecting the rise of the gluon momentum distribution in this region. This is because the hard-Pomeron exchange defined by DL model is expected to hold in the small- x limit. Also we compared the longitudinal structure function with

on-shell limit of the k_T factorization, and with the leading twist-2 term from the dipole picture. Comparing our results at NNLO with the QCD predictions from other sets is good and these results are consistent with previous observations [5,28,43-45,56].

In conclusion, we have computed the longitudinal structure function based on the Laplace transforms at low- x . These calculations allow us to determine the gluonic longitudinal structure function at small- x , directly from the initial distribution at $Q^2 = Q_0^2$ where Q_0^2 is the starting value for the evolution. The calculations are consistent with the experimental data for H1 collaboration. We compared our result with the k_T factorization scheme in the collinear and the dipole limits. As an illustration of this paper, we have used the analytical solution to the evolution equation to obtain test of the consistency of published longitudinal structure function and predict these results to small- x and high- Q^2 values as compared with the recently data from H1 Collab. at $Q^2 = 200 \text{ GeV}^2$.

References

1. A.Gonzalez-Arroyo, C.Lopez, and F.J.Yndurain, *phys.lett.***B98**, 218(1981).
2. A.M.Cooper- Sarkar, G.Inglman, K.R.Long, R.G.Roberts, and D.H.Saxon , *Z.Phys.***C39**, 281(1988).
3. R.G.Roberts, *The structure of the proton*, (Cambridge University Press 1990)Cambridge.
4. S.Aid et.al, *H1 collab.* *phys.Lett.* **B393**, 452-464 (1997).
5. R.S.Thorne, *phys.Lett.* **B418**, 371(1998); arXiv:hep-ph/0511351(2005).
6. C.Adloff et.al, *H1 Collab.*, *Eur.Phys.J.***C21**, 33(2001).
7. N.Gogitidze et.al, *H1 Collab.*, *J.Phys.***G28**, 751(2002).
8. A.V.Kotikov and G.Parente, *JHEP* **85**, 17(1997).
9. A.V.Kotikov and G.Parente, *Mod.Phys.Lett.***A12**, 963(1997).
10. R.D.Ball and S.Forte, *Phys.Lett.***B77**, 336(1994).
11. A. De Rujula, S.L. Glashow, H.D. Politzer, S.B. Treiman, F. Wilczek and A. Zee, *Phys. Rev.* **D10**, 1649 (1974).
12. Yu.L. Dokshitzer, *Sov. Phys. J.E.T.P.***46**, 641(1977).
13. S.Catani, *Z.Phys.***C75**, 665(1997).
14. A.Vogt, *Comp.Phys.Comm***170**, 65(2005).
15. G.Altarelli and G.Martinelli, *Phys.Lett.***B76**, 89(1978).
16. Yu.L.Dokshitzer, *Sov.Phys.JETP* **46**, 641(1977).
17. G.Altarelli and G.Parisi, *Nucl.Phys.B* **126**, 298(1977).
18. V.N.Gribov and L.N.Lipatov, *Sov.J.Nucl.Phys.* **15**, 438(1972).
19. M.M.Block, L.Durand, D.W.McKay, *Phys.Rev.***D77**, 094003(2008).
20. M.M.Block, L.Durand, D.W.McKay, *Phys.Rev.***D79**, 014031(2009).
21. M.M.Block, *Eur.Phys.J.***C69**, 425(2010).
22. M.M.Block, L.Durand, P.Ha and D.W.McKay, *Phys.Rev.***D83**, 054009(2011).
23. M.M.Block, L.Durand, P.Ha and D.W.McKay, *Eur.Phys.J.***C65**, 1(2010).
24. M.Hentschinski, A.Sabio Vera and C.Salas, *Phys.Rev.***D87**, 076005(2013).
25. M. Hentschinski and A. Sabio Vera, *Phys. Rev. D* **85**, 056006(2012).
26. D.I.Kazakov, et.al., *Phys.Rev.Lett***65**, 1535(1990).
27. J.L.Miramontes, J.sanchez Guillen and E.Zas, *Phys.Rev.D* **35**, 863(1987).
28. S.Moch, J.A.M.Vermaseren, A.vogt, *Phys.Lett.B* **606**, 123(2005).
29. A.D.Martin, W.J.Stirling, R.S.Thorne, *Phys.Lett.B* **635**, 305(2006).
30. A.D.Martin, W.J.Stirling, R.S.Thorne, *Phys.Lett.B* **636**, 259(2006).
31. B.G. Shaikhatdenov, A.V. Kotikov, V.G. Kriukhizhin, and G. Parente, *Phys.Rev.***D81**, 034008(2010).
32. G.G.Callan and D.Gross, *Phys.Lett.***B22**, 156(1969).
33. E.B.Zijlstra and W.L Van Neerven, *Nucl.Phys.***B383**, 552(1992).
34. G.R.Boroun and B.Rezaei, *Eur.Phys.J.***C73**, 2412(2013).
35. G.R.Boroun and B.Rezaei, *Eur.Phys.J.***C72**, 2221(2012).
36. S.Moch, J.Vermaseren and A.Vogt, *Nucl.Phys.B* **688**, 101(2004).
37. S.Moch, J.Vermaseren and A.Vogt, *Nucl.Phys.B* **691**, 129(2004).
38. A.Retey, J.Vermaseren , *Nucl.Phys.B* **604**, 281(2001).
39. A. Donnachie and P.V.Landshoff, *Phys.Lett.***B533**, 277(2002).
40. A. Donnachie and P.V.Landshoff, *Phys.Lett.***B550**, 160(2002).
41. J.R.Cudell, A. Donnachie and P.V.Landshoff, *Phys.Lett.***B448**, 281(1999).
42. P.V.Landshoff, arXiv:hep-ph/0203084.
43. A.D.Martin, R.G.Roberts, W.J.Stirling,R.S.Thorne, *Phys.Lett.B* **531**, 216(2002).
44. A.D.Martin, R.G.Roberts, W.J.Stirling,R.S.Thorne, *Eur.Phys.J.***C23**, 73(2002).
45. A.D.Martin, R.G.Roberts, W.J.Stirling,R.S.Thorne, *Phys.Lett.B* **604**, 61(2004).
46. F.D. Aaron, et al., [H1 Collaboration], *Eur.Phys.J.***C71**, 1579(2011).
47. F.D. Aaron, et al., [H1 Collaboration], *Phys.Lett.***B665**, 139(2008).
48. V. Andreev, et al., [H1 Collaboration], arXiv:1312.4821v1 [hep-ex](2013).
49. K.Golec-Biernat and A.M.Stasto, *Phys.Rev.***D80**, 014006(2009).
50. S.Catani, M.Ciafaloni and F.Hautmann, *Phys.Lett.***B242**, 97(1990).
51. J.C.Collins and R.K.Ellis, *Nucl.Phys.***B360**, 3(1991).

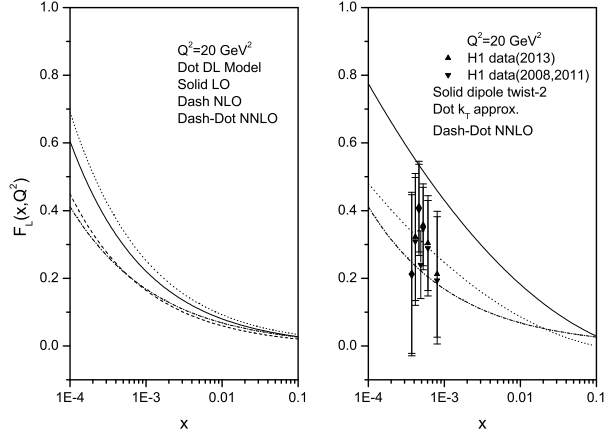


FIG. 1: Predictions for $F_L^g(x, Q^2)$ at $Q^2 = 20 \text{ GeV}^2$ at LO up to NNLO, compared with H1 data [46-48], DL model [39-42], dipole twist-2 [49,53-55] and k_T approximation [49-52].

52. J.Kwiecinski, A.D.martin and A.M.Stasto, Phys.Rev.D**56**, 3991(1997).
53. J.Bartles, K.Golec-Biernat and K.Peters, Eur.Phys.J.C**17**, 121(2000).
54. K.Golec-Biernat and M.Wusthoff, Phys.Rev.D**59**, 014017(1999).
55. J.Bartles, K.Golec-Biernat and L.Motyka, Phys.Rev.D**81**, 054017(2010).
56. C.Pisano, arXiv:hep-ph/0810.2215.

TABLE I: The QCD coupling and corresponding Λ parameter for $n_f = 4$, for LO, NLO and NNLO fits according to Ref.[43-45].

	$ \alpha_s(M_Z^2) $	$ \Lambda_{QCD}(MeV) $
LO	0.130	220
NLO	0.119	323
NNLO	0.1155	235

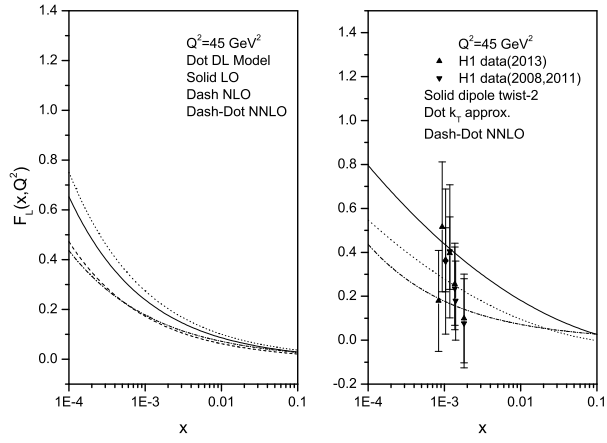


FIG. 2: The same as Fig.1 at $Q^2 = 45 \text{ GeV}^2$.

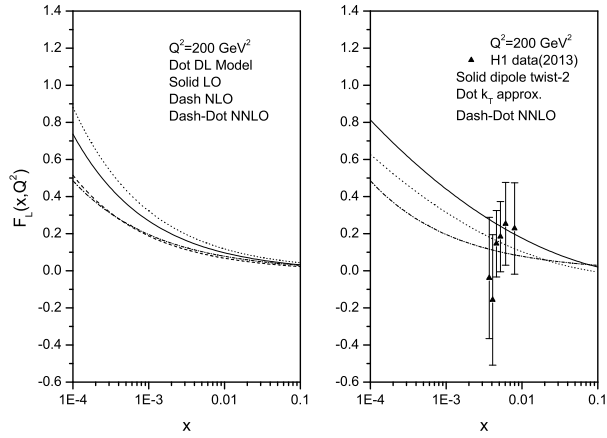


FIG. 3: The same as Fig.1 at $Q^2 = 200 \text{ GeV}^2$.